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Towards clarity on understanding tropical cyclone intensification

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Abstract:

We review an emerging paradigm of tropical cyclone intensification in the context of the prototype intensification problem, which relates to the spin up of a pre-existing vortex near tropical storm strength in a quiescent environment. In addition, we review briefly what is known about tropical cyclone intensification in the presence of vertical wind shear. We go on to examine two recent lines of research that seem to offer very different views to understanding the intensification problem. The first of these proposes a mechanism to explain rapid intensification in terms of surface pressure falls in association with upper level warming accompanying outbreaks of deep convection. The second line of research explores the relationship between the contraction of the radius of maximum tangential wind and intensification in the classical axisymmetric convective ring model, albeit in an unbalanced framework. We challenge a finding of the second line of research that appears to cast doubt on a recently suggested mechanism for the spin up of maximum tangential wind speed in the boundary layer, a feature seen in observations. In doing so we recommend some minimum standards required for a satisfactory explanation of tropical cyclone intensification.

KEY WORDS Tropical cyclone, hurricane, typhoon, spin-up, intensification, rotating deep convection

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1 Introduction

Because of the challenges of forecasting tropical cyclone intensity change, the problem of understanding how intensity change occurs has been at the forefront of tropical cyclone research for a number of years, especially in the context of the rapid intensification or decay of storms. Rapid intensification (often abbreviated RI) is conventionally defined as an increase in near-surface 1-minute average wind speed exceeding about 15 m s^{-1} over a period of 24 h (Kaplan and DeMaria 2003), although it seems unlikely that there is anything particularly special about this threshold and unlikely also that there is a fundamental difference between the physical processes of "intensification" and "rapid intensification". Presumably, there are simply quantitative differences in the strength of processes that are contributing to intensification, such as the vigor and persistence of deep convection in a region, in relation to those processes that are trying to thwart it, such as vertical wind shear or cooler water temperatures. The foregoing view is contrary to that expressed in some studies to be discussed herein, which have offered explanations of rapid intensification as if there is something fundamentally special about it.

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As elegantly articulated by Davis and Emanuel (1991): "Our ability to forecast cyclone and anticyclone behaviour has increased to the point where accurate forecasts of two days and longer are routine. However, a proper integration of the equations of motion is not synonymous with a conceptual grasp of the phenomena being predicted. Indeed, the emphasis on forecasting may have contributed to an unhealthy separation between observational and theoretical work on cyclone dynamics. Observations and theory have yet to be reconciled on some important topics and there has not been enough work to separate the underlying physics of cyclone development from unsystematic details of individual cases. These are necessary if a simple conceptual picture of cyclogenesis is to emerge. A conceptual understanding is not only useful for reconciling theory with observation, but it is valuable also for delineating measurements necessary for accurately integrating forecast models." While these remarks were aimed at middle-latitude cyclones and anticyclones, they are appropriate also when applied to tropical cyclones.

In this essay we seek to articulate what is known about tropical cyclone intensification and to expose concerns with some recent arguments that appear to fall outside of "conventional wisdom". In particular, we suggest some minimum standards required of a satisfactory explanation of tropical cyclone intensification in particular circumstances. We hope that our essay will help provide an improved foundation for analysing both observations and numerical simulations of tropical cyclone behaviour.

The paper is organized as follows. We begin in section 2 with a brief summary of what is known about tropical cyclone intensification in a relatively quiescent environment. Then, in section 3 we discuss what is known about the hostile role of vertical shear. In section 4 we examine a number of purported explanations for rapid intensification. Finally, in section 5 we appraise an attempted revival of the axisymmetric convective ring model for intensification. The conclusions are given in section 6.

2 What we know

In a recent review paper (Montgomery and Smith 2014), the authors examined and compared four paradigms for tropical cyclone intensification in the prototype problem for intensification. This problem relates to the evolution of a prescribed, initially cloud free, axisymmetric, baroclinic vortex in a quiescent environment over a warm ocean on an f-plane. The quiescent environment has served historically as the prototype configuration for understanding basic aspects of tropical cyclone intensification not involving strong interactions with the storm environment. The four paradigms reviewed are: (1) the CISK[1] paradigm; (2) the cooperative intensification paradigm; (3) a thermodynamic air-sea interaction instability paradigm (widely known as WISHE[2]); and (4) a new rotating convection paradigm (see Montgomery and Smith 2014 for references).

The first three paradigms assume axisymmetric flow (no departures from axial symmetry about the vortex rotation axis, i.e., no azimuthal eddies). A recent investigation by Persing et al. (2013) suggests that previous studies using strictly axisymmetric models, and their attendant phenomenology of axisymmetric convective rings, have intrinsic limitations for understanding the intensification process. Particular problems with the CISK paradigm are discussed in Montgomery and Smith (2014). Futhermore, calculations presented by Montgomery et al. (2009) and more recently by Montgomery et al. (2014a) show that the WISHE paradigm, currently the most widely cited intensification paradigm, is not the dominant mode of intensification in the prototype problem.

The new paradigm is intrinsically three-dimensional and recognizes the presence of localized, rotating deep convection that grows in the cyclonic rotation-rich environment of the incipient storm. The updrafts within these convective structures greatly amplify the vorticity locally by vortex-tube stretching and the patches of enhanced cyclonic vorticity subsequently aggregate to form a central monolith of cyclonic vorticity. The mean field dynamics of the rotating convection paradigm constitute an extended cooperative intensification paradigm in which eddy processes can contribute positively to amplifying the tangential winds of the vortex. In this azimuthally-averaged view, illustrated schematically in Figure 1, there are two mechanisms for spin up besides the eddy processes.

2.1 Conventional spin up mechanism

The first mechanism is a key element of the cooperative intensification paradigm in which the spin up of the winds above the boundary layer (that are widely held to be in approximate gradient wind balance) is accomplished by the convectively-induced inward radial advection of the surfaces of absolute angular momentum[3], M, where this quantity is approximately materially conserved. It is assumed that surface moisture fluxes are sufficient to maintain the required deep convective activity.

2.2 Boundary-layer spin up mechanism

Perhaps counter intuitively, the spin up of the maximum tangential winds takes place within the frictional boundary layer, where M is not materially conserved and where the winds are no longer in approximate gradient wind balance. The breakdown of gradient wind balance by the frictional retardation of the tangential wind component leads to a net inward force in the boundary layer and, as it turns out, to a much stronger inflow than in the vortex above. The stronger the inflow, the shorter is the trajectory of air parcels as they spiral inwards and therefore the smaller is the loss of M caused by the frictional torque. Spin up of the maximum tangential winds in the boundary layer is possible if the fractional rate of reduction of M is less than the fractional rate of reduction of inward displacement for an air parcel. The two mechanisms of spin up are coupled through boundary layer dynamics. Moreover, a spin up of the winds in the boundary layer requires a spin up of the winds above the boundary layer as well. The foregoing ideas provide an explanation for observations that the maximum storm-relative tangential winds occur in the boundary layer (Kepert 2006a,b; Montgomery et al. 2006; Schwendike and Kepert 2008; Sanger et al. 2014; Montgomery et al. 2014b).

2.3 Coupling, ventilation

From an azimuthally-averaged perspective, in the absence of convective forcing, the frictionally-induced inflow within the boundary layer would be accompanied by a shallow[4] layer of outflow above the boundary layer and, by the material conservation of M in this outflow, to a spin down of the vortex. This spin down would be accompanied, through approximate gradient wind balance, by a demise of the radial pressure gradient at the top of the boundary layer. This process of vortex spin down was articulated by Greenspan and Howard (1963) and was examined in the

[1] Conditional Instability of the Second Kind.
[2] Wind Induced Surface Heat Exchange.

[3] The quantity M is defined in terms of the tangential wind speed v by the formula $M = rv + \frac{1}{2}fr^2$, where r is the radius and f is the Coriolis parameter. Alternatively, $v = M/r - \frac{1}{2}fr$.
[4] Shallow because the atmosphere is stably stratified.

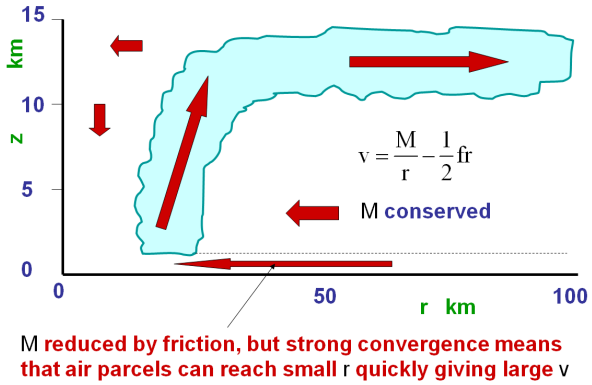

Figure 1. Schematic of the axisymmetric view of tropical cyclone intensification in the new paradigm. Above the boundary layer, spin up of the vortex occurs as air parcels are drawn inwards by the inner-core convection. Air parcels spiralling inwards in the boundary layer may reach small radii quickly (minimizing the loss of absolute angular momentum, M, during spiral circuits) and acquire a larger tangential wind speed v than that above the boundary layer.

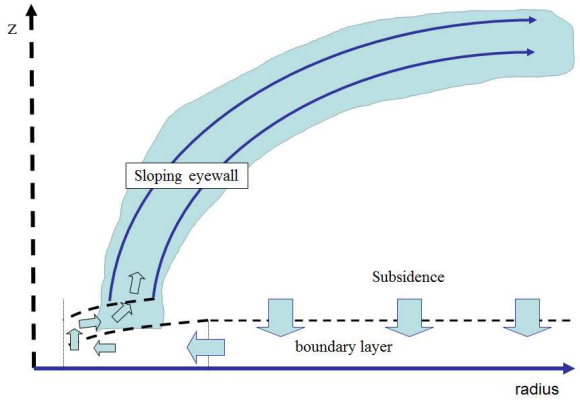

Figure 2. Schematic of the hurricane inner-core region in relation to the broader scale overturning circulation. Air subsides into the boundary layer at large and moderate radii and ascends out of the boundary layer at inner radii. The frictionally-induced net inward force in the boundary layer produces a radially inward jet. The subsequent evolution of this jet depends on the bulk radial pressure gradient that can be sustained by the mass distribution at the top of the boundary layer. The jet eventually generates supergradient tangential winds whereafter the radial inflow rapidly decelerates. As it does so, the boundary layer separates and the flow there turns upwards and outwards to enter the eyewall. As this air ascends in the eyewall, the system-scale tangential wind and radial pressure gradient come into mutual balance. This adjustment region has the nature of an unsteady centrifugal wave with a vertical scale of several kilometres.

hurricane context by Eliassen (1971), Eliassen and Lystad (1977), and Montgomery et al. (2001). Clearly, for a vortex to spin up, the convectively-induced inflow must be sufficient to outweigh the frictionally-induced outflow above the boundary layer. In other words, the convection itself must be strong enough to more than "ventilate" the mass converging in the boundary layer associated with friction, it must be strong enough to produce inflow above the boundary layer also.

2.4 Role of asymmetric eddies

Persing et al. (2013) demonstrated that, within the new intensification paradigm, eddy processes can contribute positively to amplifying the tangential winds of the vortex. This positive contribution to vortex spin up contrasts with previous assumptions and speculation of the downgradient action of asymmetric motions (referred to as "turbulence, but including vortical convection and vortex Rossby waves and their wave-mean-flow and wave-wave interactions), which would lead to spin down (Bryan et al. 2009). The findings of Persing et al. (2013) suggest that previous studies using strictly axisymmetric models and their attendant phenomenology of axisymmetric convective rings have intrinsic limitations for understanding the intensification process.

2.5 Balance dynamics and its limitations

As is well known, approximations to one or more of the governing equations for tropical cyclone evolution may be invoked to simplify the problem. In fact, guided by a scale analysis of the azimuthally-averaged equations for the bulk flow about the storm centre expressed in cylindrical coordinates, it is frequently assumed that the system-scale vortex is approximately in hydrostatic and gradient wind balance (e.g. Willoughby 1979). For an axisymmetric vortex, these assumptions constrain the primary (or tangential) circulation above the boundary layer to be in thermal wind balance at all times. This constraint determines an equation for the streamfunction of the secondary (or overturning) circulation, which is required to maintain balance in the presence of processes trying to drive the system away from balance. Such processes include radial and vertical gradients of diabatic heating associated with latent heat release in deep convection, or vertical gradients of any frictional force in the boundary layer. Because of the pioneering work of Eliassen (1951, 1962) and Sawyer (1956) for both circular vortex and frontal circulations, the equation for the overturning circulation is often referred to as the Sawyer-Eliassen equation. When combined with the remaining unapproximated component of the momentum equations, i.e. that for the tangential component, one can develop a prognostic system of equations[5] governing the evolution of a balanced vortex when the forcing terms in the Sawyer-Eliassen equation are prescribed or parameterized.

The balance theory does not strictly apply to a steady state vortex because the derivation of the Sawyer-Eliassen equation is formally not possible in this case. However, the existence of a realistic globally-steady state for a tropical cyclone has been questioned recently (Smith et al. 2014b). For one thing, such a state would require a steady supply of

[5]There are, of course, technical issues that can arise in the solution of the Sawyer-Eliassen equation in localized regions where the equation ceases to be elliptic (Möller and Shapiro 2002; Bui et al. 2009).

cyclonic relative angular momenum to replenish that lost to the system by friction.

One limitation of the balance theory described above is that it is neither accurate nor formally applicable in the boundary layer, where the assumption of gradient wind balance breaks down. In principle one might think of applying the theory above the boundary layer and using a *nonlinear* boundary layer model to predict the radial profiles of vertical velocity and thermodynamic quantities at the top of the boundary layer. However, this approach has its own problems because of the separation of the boundary layer beneath the eyewall and the fact that the air being lofted into the eyewall is not generally in gradient wind balance and must adjust to balance (albeit not to a prescribed balance state) as it rises into the eyewall. This adjustment has the form of a centrifugal wave, which model simulations show to be typically unsteady. Other aspects of this inertially-dominated corner flow and its interaction with deep convection are discussed in section 5. A schematic of the corner flow region is shown in 2.

Explicit comparisons between two different full physics mesoscale models and the Sawyer-Eliassen model and corresponding tangential wind tendency have been carried out in Smith et al. (2009) and Abarca and Montgomery (2014). These studies have shown that during vortex spin up the radial inflow in the boundary layer region using the balance model was insufficient to offset the frictional spin down effect. In other words, the balance model cannot capture the spin of the tangential wind in the boundary layer as observed in the full physics models.

Recent work has challenged the view that unbalanced dynamics within the boundary layer are an important aspect of the spin up of tropical cyclones, a challenge that we refute in section 5.

2.6 Geopotential tendency equation, heating efficiency

A related approach to the balance formulation just summarized is that based on the geopotential tendency equation (Shapiro and Montgomery 1993; McWilliams et al. 2003; Vigh and Schubert 2009; Persing et al. 2013). In particular, Vigh and Schubert (2009) demonstrated analytically that the surface pressure fall in the balance model is significantly larger when the imposed ring of diabatic heating lies inside the high vorticity region of the inner core. This finding reaffirmed the idea that heating within regions of high inertial stability is highly efficient for tropical cyclone spin up (Schubert and Hack 1982). However, it should be pointed out that, irrespective of the efficiency argument, a ring of convection located at an inner radius has the potential to converge air parcels to a smaller radius than a ring located at an outer radius, Thus the inner ring would be able to draw M surfaces to a smaller radius than the outer one leading potentially to a more intense vortex both above and within the boundary layer.

2.7 Applications, need for consistency

The new intensification paradigm has already proved useful in understanding the latitudinal dependence of the intensification rate for the prototype problem (Smith et al. (2014a)) and we would argue that aspects of it provide a useful starting point for understanding intensification in more complex environments with a background flow. The foregoing Smith *et al*. study highlighted the fact that any interpretation of vortex evolution, even in the absence of an environmental flow, requires consistent consideration of all three components of Newton's equations of motion, constrained by a mass continuity equation, as well as a thermodynamic equation and possibly equations for species of water substance. Explanations of intensification that fail to consider any one or more of these equations must be viewed with suspicion.

Arguments based on the balance formulation provide a succinct means for providing an *understanding* of the evolution of vortex structure, at least in an axisymmetric or weakly asymmetric framework, but it is imperative that these arguments remain within the framework of the theory. As an example, for the axisymmetric problem, it is no longer valid to invoke imbalances of forces in the radial or vertical directions as part of such arguments.

Because of the tight coupling between the boundary layer and the vortex above it, the construction of cause and effect arguments to explain vortex behaviour is fraught with danger. For example, as noted earlier, the boundary layer dynamics and thermodynamics control the radial profiles of radial and tangential velocity components within the boundary layer as well as those of vertical velocity, horizontal momentum and equivalent potential temperature that exit its top into the eyewall. It is the radial profiles of vertical velocity and equivalent potential temperature that determine[6], in part, the radial gradient of diabatic heating rate in the eyewall. In turn, it is the radial and vertical gradients of diabatic heating rate as well as the forcing from the vertical velocity at the top of the boundary layer that determine the balanced secondary circulation in the vortex above the boundary layer. It is the inward branch of this circulation that determines, *inter alia*, changes to the tangential wind profile at the top of the boundary layer which then feeds back to determine the flow in the boundary layer, itself. Finally, the thermodynamics of the boundary layer are controlled in the outer region by the subsidence of vortex air into the boundary layer and the surface enthalpy flux, which depends, in part, on the surface wind speed (see e.g. Smith and Vogl 2008).

If one doesn't invoke balance dynamics, any viable theory for intensification needs to consider *all* the governing equations. Of course, balance dynamics as defined above cannot be formally justified in the boundary layer

[6]The diabatic heating rate, $\dot{Q} = D\theta/Dt$, is approximately related to the vertical velocity, w and equivalent potential temperature θ_e by the formula $\dot{Q} = \mu w$, where $\mu = -L(\partial q_v/\partial z)_{\theta_e = constant})$, where L is the latent heat of condensation and q_v is the water vapor mixing ratio.

and there are important physical issues in coupling the boundary layer to the interior flow (Smith et al. 2008; Smith and Montgomery 2010; Abarca and Montgomery 2014).

3 Intensification in hostile environments: what we know about the effects of vertical shear

In some real-world cases of tropical-cyclone intensification, the ambient flow is not weak and the magnitude of the vertical shear impinging on a storm is one of the critical parameters sought by forecasters. Although the new paradigm should still provide a useful building block for understanding vortex spin up in these more complex circumstances, we would expect that important modifications to it would emerge in the form of coherent eddy processes associated with the interaction of the vortex with the impinging vertical shear (Reasor et al. 2004; Reasor and Montgomery 2014, and refs.) and their coupling to the boundary layer and convection, as well as the projection of these eddy processes on the azimuthally-averaged vortex dynamics.

A recent study of tropical cyclogenesis in wind shear by Nolan and McGauley (2012) gives a review of five decades of empirical and numerical modeling research examining the effects of vertical and horizontal wind shear on tropical cyclogenesis. The paper discusses also a suite of new numerical experiments and diagnostic analyses for a sheared vortex undergoing genesis and intensification. Although the tropical cyclogenesis problem lies beyond the scope of the present paper, Nolan and McGauley give modelling results and insightful interpretations that would appear to apply to the intensification problem as well, after genesis occurred[7]. One of Nolan and McGauley's principal conclusions was that large vertical shear values "delayed or suppressed further development (intensification after genesis, our insertion), consistent with a substantial body of previous work regarding the effects of wind shear on developing and mature tropical cyclones (Frank and Ritchie 2001; Wong and Chan 2004; Riemer et al. 2010; DeMaria and Kaplan 1999; Tang and Emanuel 2010)".

Insofar as the role of vertical shear in the intensification problem, the emerging view from Nolan and McGauley (2012) and complementary theoretical work is that moderate or weak vertical shear excites new dynamic- thermodynamic pathways through which relatively dry air may be entrained into the moist envelope region of the vortex. These pathways act to generate mesoscale downdrafts that flush portions of the boundary layer with low-level moist equivalent potential temperature (θ_e) air that originate from above the boundary layer near the minimum of θ_e and outside of the moist envelope (Tang and Emanuel 2010; Riemer et al. 2010, 2013; Riemer and Montgomery 2011). A boundary layer with reduced θ_e suppresses convective instability, and, unless the moisture fluxes can ameliorate the θ_e deficit for inward-spiraling air parcels, the vortex will begin to spin down until the boundary layer can recover to its pre-shear values and intensification can resume (Riemer et al. 2010, 2013).

The foregoing is a broad-brush synthesis of what we know about the physical effects of vertical shear in tropical cyclone intensification. Without a doubt, the vertical shear intensification problem is an important scientific problem of societal relevance and further basic research on it is clearly warranted to better understand the dynamic-thermodynamic pathways that have been discovered in recent work. However, before embarking on a systematic programme, it is critical to have a solid understanding of the intensification problem in the prototype problem as defined above.

4 An appraisal of a recent upper-level warming hypothesis

A series of recent papers has purported to articulate a mechanism for the rapid intensification of hurricanes (Chen et al. 2011; Zang and Chen 2012; Chen and Zhang 2013; Chen and Gopalakrishnan 2014). The basis of the theory appears to be encapsulated in Figure 1b of Chen and Zhang (2013), which shows time series of minimum surface pressure from a simulation of Hurricane Wilma (2005), obtained by integrating the hydrostatic equation from the model top downwards using the temperature and the "δlnp-weighted warming" as defined precisely in the caption of their figure. They show that there is an inverse relationship between the evolution of the minimum mean sea level surface pressure and that of the weighted warming. The surface pressure fall with time hinges on the idea that, from the hydrostatic equation " ... a higher-level warm core will *cause* (our emphasis) a greater surface pressure fall than a lower-level one because of the more amplifying effects of the upper level warming". They say that " ... the warm core in the eye results from the detrainment of CBs (convective bursts, our insertion) that occur mostly in the vicinity of the RMW (radius of maximum wind, our insertion) where higher equivalent potential temperature is located. Then, the CBs' detrainment (*sic*) enhances collectively cyclonic radial inflows above the upper-level outflow layer that are associated with the mass sink in the eye, leading to the subsidence warming below with the peak intensity occurring in the same layer as the upper-level outflow."

4.1 Some concerns

We have a number of questions about the proposed mechanism that we could not find answers to within these papers. For example, there appears to be no discussion of how the diagnosed pressure falls are linked dynamically to changes

[7]There are supporting theoretical reasons to believe that a unified view of the genesis and intensification problems is meaningful (Montgomery and Smith 2011; Riemer and Montgomery 2011).

in wind speed. Based on this proposed mechanism, it would seem that the problem of hurricane rapid intensification is to be understood simply in terms of hydrostatic reasoning in combination with the kinematics of convective bursts and the accompanying temperature effects of these bursts. The connection between the pressure fall and the increase of the tangential wind is not explained. We attempt now to appraise details of the proposed mechanism.

First, while it is true that if a temperature perturbation of fixed magnitude and shape in a column of air is at a higher altitude than in another case for the same temperature perturbation, and if the pressure at some finite[8] height above the temperature perturbation is the same for both situations, then the surface pressure in the case with the higher temperature perturbation will be lower. Nevertheless, there remains the question as to why the pressure at the height above the temperature perturbation should be the same in both situations. Moreover, attempts to explain the *rate-of-change* of surface pressure accompanying a particular distribution of warming requires one to explain how the mass within the column is reduced to lower the surface pressure. Strictly speaking, in a quasi-static formulation of the problem in which the vertical momentum balance is simply hydrostatic balance, then if one takes the partial time derivative of surface pressure and uses the continuity equation one sees immediately that one is going to have to solve all of the equation to determine the amount of mass being removed. This mass removal will generally require an acceleration of fluid parcels in the two horizontal directions.

Clearly, the motions that accomplish the evacuation of mass within the column must satisfy Newton's equations of motion! For example, for a thermally-forced vortex evolving slowly in hydrostatic and gradient wind balance, the evacuation of mass is governed by the solution of the Sawyer-Eliassen equation with the appropriate radial and vertical gradients of the diabatic heating rate appearing in forcing terms on the right-hand-side of this equation (see e.g. Shapiro and Willoughby 1982; Montgomery and Smith 2014). An even more direct explanation for the surface pressure fall is contained within the framework of the geopotential tendency equation that is forced by the spin-up[9] function (Shapiro and Montgomery 1993; Vigh and Schubert 2009; Persing et al. 2013). In either case one still has to use a dynamical equation of motion for the rotational flow.

4.2 Confusion over the role of WISHE

For the foregoing reasons, the statement by Chen and Zhang (2013) that: "Later, Zang and Chen (2012) showed that the model predicts an intense warm core in the same layer as the upper-level outflow at the time of peak intensity and *then demonstrated that* (our emphasis) this upper-level warm core is responsible for most of the RI of Wilma", would seem to be unsupported. Chen and Zhang (2013) state further that " ... the above-mentioned results are not surprising, based on the wind-induced surface heat exchange (WISHE) theory that was first discussed by Ooyama (1969) and later clarified by Emanuel (1986, 1991) and Rotunno and Emanuel (1987). However, the WISHE theory does not relate the roles of SSTs in RI to *the efficiency of the upper-level warm core* (our emphasis)." They say also that "RI is determined by SSTs through the WISHE process and active convective bursts in inner-core region that penetrate to high altitudes." Although Chen and Zhang do not define what they mean by the WISHE process in their paper, it is our understanding that the process is based on a model in which the cloud buoyancy was effectively zero and requires a wind-speed dependent flux of moisture. Moreover, the process was first coined in a paper by Yano and Emanuel (1991), so that its attribution to Ooyama is a little surprising (cf. Ooyama (1997)). A recent appraisal of the hypothesized WISHE mechanism for the prototype intensification problem is given by Montgomery et al. (2014a).

4.3 Chen and Gopalakrishnan 2013

In another recent paper, Chen and Gopalakrishnan (2014) presented the results of a forecast from the operational Hurricane Weather Research and Forecasting (HWRF) system for Hurricane Earl (2010). In their paper the forecast was broadly verified against a multitude of aircraft observations from a joint NOAA-NASA field experiment, with the goal to understand the asymmetric rapid intensification of a storm in a sheared environment. They conclude that "the RI onset is associated with the development of upper-level warming in the eye, which results from upper-level storm-relative flow advecting the warm air caused by subsidence warming in the upshear-left region towards the low-level storm center." Apparently, this process "does not occur until persistent convective bursts (CB) are concentrated in the downshear-left quadrant." At this stage, "the subsidence warming is maximized upshear and then advected towards the low level storm center by the storm-relative flow at the upper level. Subsequently, the surface pressure falls and RI occurs." The main difference between the envisaged mechanism and that proposed by Chen and Zhang (2013) for the case of Hurricane Wilma appears to be the role of horizontal temperature advection in "advecting the warm air caused by subsidence warming in the upshear-left region towards the low-level storm center."

Our concerns with this study are the same as with the earlier ones discussed above: there is no consideration given to dynamical processes.

[8] It is well known that the surface pressure in a column in hydrostatic balance is determined by the vertical integral of the density multiplied by gravity over the atmosphere. However, the surface pressure is not determined by the vertical distribution of the temperature alone unless the pressure at some finite level is known.

[9] cyclogenesis function

5 A retreat to the axisymmetric ring model

In a recent paper, Stern et al. (2014) have challenged an aspect of what they consider to be a widely-held viewpoint that the intensification of tropical cyclones is accompanied by the contraction of the RMW. They showed that in idealized numerical simulations using the WRF model, "contraction and intensification commence at the same time, but that contraction ceases long before peak intensity is achieved." They pointed out that "in the convective ring model, it is the secondary circulation induced by condensational heating that causes contraction, as the associated tangential wind tendency is maximized inwards of the RMW." The convective ring model is based the balance dynamics of an axisymmetric vortex forced by axisymmetric heat and momentum sources (Shapiro and Willoughby 1982; Willoughby 1979, 1995). We are puzzled by their focus on the contraction of the RMW since it is the movement of the absolute angular momentum surfaces that are more fundamental to understanding intensification, and as Stern *et al*. admit, there is not a one-to-one relationship between these surfaces and the RMW. Even if the RMW ceases to contract, an inward movement of the absolute angular momentum surfaces *must* accompany vortex intensification.

In their section 4, Stern *et al*. re-examine the convective ring model using the time dependent linear vortex model (3DVPAS) of Nolan and Montgomery (2001) and Nolan and Grasso (2003). This model was developed originally to examine linear asymmetric disturbances and their wave, mean flow interactions in hurricanes and tornado vortices. Stern *et al*. use this dry, non-hydrostatic model to solve for the azimuthal wavenumber zero perturbation flow and corresponding tangential wind tendency to the assumed steady vortex under the imposed forcing. They say that this methodology is "similar (in result) to solving a diagnostic Sawyer-Eliassen equation, which has been used in a number of studies (e.g. Bui et al. (2009))", noting that "3DVPAS yields a very similar result to the analytical solutions of the Sawyer-Eliassen equation given in Schubert et al. (2007) and Rozoff et al. (2008)". While the test they used to compare the secondary circulation predicted by the linearized model with that predicted by the Sawyer-Eliassen equation is fine for the flow above the boundary layer, it cannot address the situation within the boundary layer, where the flow is intrinsically nonlinear (Prandtl 1904; Smith 1968; McWilliams 1971; Anderson 2005; Vogl and Smith 2009), and where separation and vortex breakdown are generally elements of the boundary layer spin up process (Smith and Montgomery 2010; Montgomery and Smith 2014; Rotunno 2014).

Stern et al. critique the results of Bui et al. (2009) and Abarca and Montgomery (2013)), who they say " ... have proposed that the spin-up of the inner core of tropical cyclones is largely a result of frictionally-driven inflow in the boundary layer, as opposed to spin-up occurring through a deep layer of heating-induced inflow". As we discussed in section 2, we specifically describe two mechanisms for spin up, the first the conventional mechanism that involves the convectively-induced inflow in association with a positive radial gradient of absolute angular momentum and the second being the mechanism associated with frictional boundary layer dynamics discussed above. Further, we point out that these two mechanisms are coupled through boundary layer dynamics. As discussed above, the new paradigm does not propose that the spin-up of the inner core of tropical cyclones is largely a result of frictionally-driven inflow alone: while we have shown that the spin up of the maximum tangential winds occurs in the boundary layer, this spin up cannot occur without spin up above the boundary layer as well[10].

Stern et al. say that " it is indeed possible to reproduce the low-level inflow in simulated TCs (tropical cyclones, our insertion) as the combined response of a balanced vortex to heating and friction, and this casts doubt on recent theories that appeal to unbalanced dynamics to explain intensification." However, we would point out that one simply cannot expect to use a linear model to assess a theory founded on intrinsically nonlinear processes (i.e. where the nonlinear terms are comparable or larger in magnitude than the linear ones). Stern et al. remark that, as far as they are aware, no study has yet to compare "the positive tendency on tangential winds due to frictional inflow with the negative tendency on tangential winds due to friction itself." The reason is presumably because of the difficulties posed by the full nonlinearity of the inner-core boundary layer.

Stern et al.'s linear model starts from a basic state in gradient and hydrostatic balance with zero radial flow and zero vertical gradient of tangential velocity below a height of 500 m, which would span a large part of the boundary layer, and solves for the evolution to perturbations to this basic state. The linearization is valid, of course, only as long as the perturbation remain sufficiently small that the neglect of the nonlinear terms is formally justified. As shown by Stern et al., radial inflow in the boundary layer and presumably the perturbation tangential wind speed will

[10] As noted in Montgomery and Smith (2014), these spin up processes were erroneously stated to be independent in Smith *et al*. (2009) and this may have confused Stern et al.. Nonetheless, Abarca and Montgomery (2013) were clear about the coupling of the second mechanism to the interior swirling flow via the radial pressure gradient at the top of the boundary layer. In particular, in their Section 6, Abarca and Montgomery (2013, p3227) used a simple time dependent slab boundary layer model to illustrate the tendency of the frictional boundary layer dynamics to progressively control the initiation of a secondary eyewall. For illustrative purposes, they used a fixed radial pressure gradient diagnosed from a mesoscale model at a time just prior to the appearance of a secondary tangential wind maximum in the model. The recent study by Menelaou et al. (2014) appears to have challenged the application of the new intensification paradigm to the problem of secondary eyewall formation and the role of the frictional boundary layer dynamics. As discussed in section 2 above, in their sensitivity experiment with heating turned off, they have simply rediscovered that a vortex without sustained forcing will spin down!

become negative near the surface and increase in magnitude because of the frictional retardation of the tangential wind. Even if the tangential wind above the boundary layer was held fixed, the boundary layer perturbations would have to become large enough to formally invalidate the linearization assumption (McWilliams (1971), Vogl and Smith (2009)). If the tangential wind speed spins up above the boundary layer, the perturbation tangential wind will be even larger and lead to a larger perturbation of the radial wind. This consideration raises the question: at what point in time does the linear integration become invalid in the sense that the nonlinear terms diagnosed from it are no longer small compared with the linear terms? It is unclear from Stern *et al*. 's results whether the time shown is within the linear regime.

We should point out that a steady nonlinear slab boundary layer model[11] can capture a frictional boundary layer structure in which the radial wind speed distribution is comparable to that of a sophisticated model like the Weather Research and Forecasting model (WRF) when the tangential wind speed at the top of the boundary layer is supplied by the latter model. However, the linear approximation to the slab boundary layer model does not satisfactorily capture such structure (e.g. compare Figures 2 and 3 in Smith et al. (2014a)).

We have argued in Smith and Montgomery (2010) and in Montgomery et al. (2014a) that boundary layer theory formally breaks down in regions of deep convection owing to the horizontal pressure gradient induced in the boundary layer by the convection. In Montgomery *et al*. (2014, section 3) we pointed out that "... as buoyant air rises in a deep convective updraft, boundary layer air is drawn toward the updraft. This "suction effect cannot be described by boundary layer equations as their parabolic nature precludes their knowledge of flow properties in the downstream direction (i.e. information is conveyed in the direction of flow only)." Our point is that a strict boundary layer model knows nothing about the convectively-induced radial pressure gradient in the boundary layer. Of course, the low-level convectively induced radial pressure gradient will strengthen the frictionally induced agradient force in the boundary layer, but because of the nonlinearity of the boundary layer it is not clear to us how to quantify the relative contributions of these effects. Clearly, it cannot be done using a linear model alone.

For all of the foregoing reasons, Stern et al.'s use of the linear model to isolate the separate the effects of diabatic heating from those of friction on the dynamics within the boundary layer has no theoretical basis, casting doubt on some of the related conclusions in their paper.

[11] Smith and Montgomery (2010), Abarca and Montgomery (2013), Smith et al. (2014a), appendix

6 Conclusions

In this paper we have tried to articulate minimum requirements for a consistent theory of tropical cyclone intensification. We have outlined what we believe to be a consistent paradigm for intensification in the prototype problem thereof, which relates to spin up in a quiescent environment. We have tried to review also what we know about the effects of vertical wind shear. Finally we have tried to review two recent lines of research on the intensification problem, one of which proposes a mechanism to explain rapid intensification in terms of surface pressure falls and the other which explores the relationship between the contraction of the radius of maximum tangential wind and intensification in an axisymmetric framework. The second line of research appears to cast doubt on an aspect of the azimuthally-averaged view of the new, three-dimensional intensification paradigm reviewed here.

We have examined these studies carefully and have expressed a series of concerns with the approaches. Our main concern with the first line of research is a complete omission of dynamical processes: nowhere is Newton's law of motion, constrained by the continuity equation, invoked to explain how the central surface pressure falls and how this fall leads to the spin up of the swirling wind. Our concern with the second line of approach, which seems to be a return to a sort of convective ring model, is threefold. We see the strong focus on the movement of the radius of maximum tangential wind as a distraction, as this radius is neither a materially conserved quantity, nor is it fundamental to the dynamics of the spin up process. In an axisymmetric framework, spin up must be tied to the inward movement of the surfaces of absolute angular momentum. Our second concern is the validity of the linear model to isolate the separate effects of diabatic heating from those of friction, which has no rigorous theoretical basis within the boundary layer of a rapidly rotating vortex. Our third concern is the use of this linear methodology to cast doubt on the boundary layer spin up mechanism.

Echoing the sentiment expressed by Davis and Emanuel, our ability to simulate a phenomenon by integrating the equations of motion is not synonymous with an elemental understanding of the phenomenon being predicted. We would argue that a minimum requirement of any acceptable theory for tropical cyclone intensification is that consideration be given to all dynamical and thermodynamic equations in a consistent manner. We see the recognition and implementation of this requirement as a way forward towards clarity in the tropical cyclone intensification problem.

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